Q: Rotate the following conic section to eliminate the *xy* term:

$$6x^2 + \sqrt{3}xy + 7y^2 - 36 = 0$$

A: The required rotation angle is given by

$$\cot 2\theta = \frac{A - C}{B} = -\frac{1}{\sqrt{3}}$$

Solving this equation, we see that

$$2\theta = -\frac{\pi}{3} \quad \text{or} \quad \theta = -\frac{\pi}{6}$$

and therefore

$$\cos\theta = \frac{\sqrt{3}}{2} \qquad \sin\theta = -\frac{1}{2}$$

To transform the conic section, substitute:

$$x = x'\cos\theta - y'\sin\theta$$
$$y = x'\sin\theta + y'\cos\theta$$

or

$$x = \frac{\sqrt{3}}{2}x' + \frac{1}{2}y'$$
$$y = -\frac{1}{2}x' + \frac{\sqrt{3}}{2}y'$$

Plugging these into the original equation gives

$$6\left(\frac{\sqrt{3}}{2}x' + \frac{1}{2}y'\right)^2 + \sqrt{3}\left(\frac{\sqrt{3}}{2}x' + \frac{1}{2}y'\right)\left(-\frac{1}{2}x' + \frac{\sqrt{3}}{2}y'\right) + 7\left(-\frac{1}{2}x' + \frac{\sqrt{3}}{2}y'\right)^2 - 36 = 0$$

Which simplifies to

$$\frac{9}{2}x^2 + \frac{6\sqrt{3}}{2}xy + \frac{3}{2}y^2 - \frac{3}{4}x^2 + \frac{\sqrt{3}}{2}xy + \frac{3}{4}y^2 + \frac{7}{4}x^2 - \frac{7\sqrt{3}}{2}xy + \frac{21}{4}y^2 - 36 = 0$$

Which simplifies to

$$\boxed{\frac{11}{2}x^2 + \frac{15}{2}y^2 - 36 = 0}$$

To see the original and rotated graphs, go to http://www.wolframalpha.com and copy-paste this line:

graph 6*x^2+sqrt(3)*x*y+7*y^2 - 36 = 0, 11/2*x^2+15/2*y^2 - 36 = 0