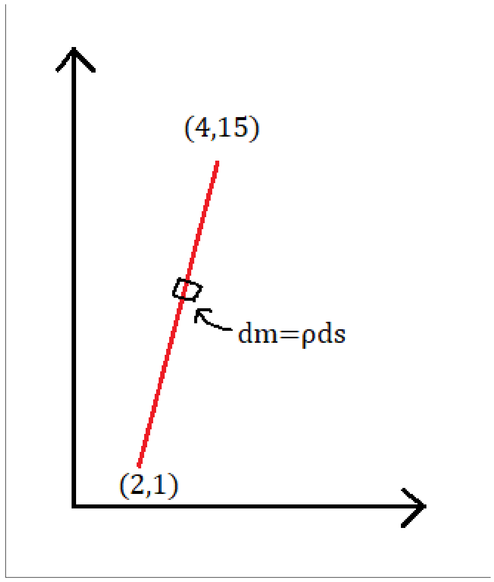
Q: Find the mass of a wire which lies on the path $(2,1) \longrightarrow (4,15)$ and whose density is given by the function $\rho(x,y) = 2xy + 7y$.



A: As one often does in calculus, we begin by breaking the wire up into an infinite number of infinitessimal mass elements
$\mathrm{d}m = \rho\,\mathrm{d}s$.

where $\mathrm{d}s$ is an infinitessimal distance along the length of the wire.

Let us paramaterize the wire by introducing a new variable t such that

$$\boldsymbol{r}(t) = \langle x(t), y(t)\rangle = \langle 2,1\rangle + t\langle 2,14\rangle$$

Notice that the values $t = 0$ and $t = 1$ correspond to the start and end of the wire, respectively. Morever we can observe

$$\mathrm{d}x = 2\,\mathrm{d}t \quad \text{and} \quad \mathrm{d}y = 14\,\mathrm{d}t$$

In terms of t the density function becomes

$$\rho(x(t), y(t)) = 2(2+2t)(1+14t) + 7(1+14t) = 11 + 158t + 56t^2$$

Meanwhile, the length element $\mathrm{d}s$ can be written as

$$\mathrm{d}s = \sqrt{\mathrm{d}x^2 + \mathrm{d}y^2} = 10\sqrt{2}\,\mathrm{d}t$$

To get the total mass M, we integrate

$$M = \int_C \rho\,\mathrm{d}s = \int_0^1 (11 + 158t + 56t^2)10\sqrt{2}\,\mathrm{d}t = \boxed{\frac{3260\sqrt{2}}{3}}$$