Lenses

In order to answer a problem involving lenses, we must know the lens equation, which takes one of the following forms:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \quad (1)$$

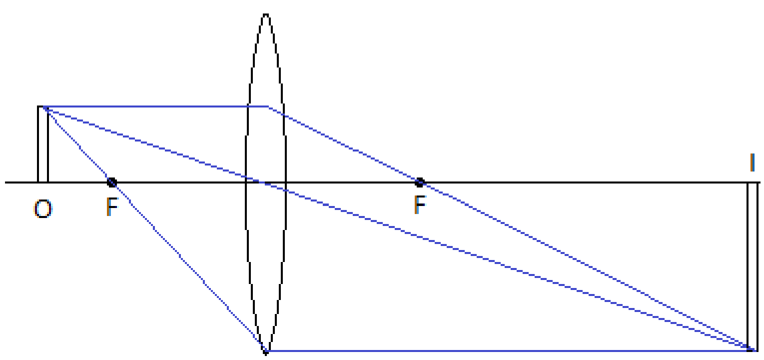
$$\frac{1}{f} + \frac{1}{d_i} = \frac{1}{d_o} \quad (2)$$

$$\frac{1}{f} + \frac{1}{d_o} = \frac{1}{d_i} \quad (3)$$

Each version of these equations corresponds to a particular configuration of lenses, objects, and images. (We will assume that each of the distances f, d_0, and d_i is positive.) Let us now examine each of these configurations in detail.

(1) Convex lens, $d_o > f$

This is probably the most common lens configuration. The result is an inverted real image that is formed beyond the other focal point. The lens functions as a sort of “relay”, carrying the light to a new location, whence the image can be reused as the object for another lens as in a microscope or telescope. The general setup looks like this:



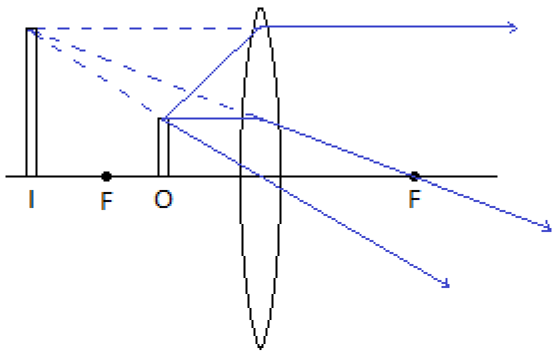
Since the disance to the focal point is less than either d_o or d_i, the reciprocal $\frac{1}{f}$ will be greater greater than either $\frac{1}{d_o}$ or $\frac{1}{d_i}$. Therefore, we must use the first version of the lens equation:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}.$$

Notice that if the object is located between f and $2f$, then the image will be magnified, while if the object is located beyond $2f$, then the image will be reduced. If the object is placed exactly at $2f$, then the object and image will be identical in size. Finally, if the object is placed exactly at the focal point, then the outgoing rays will be parallel, so no image will be formed.

(2) Convex lens, $d_o < f$

The second configuration the also uses a convex mirror, but the object is located between the lens and the focus. The results in an upright virtual magnified image (for a viewer located to the right of the lens). The most familiar use of this configuration is a magnifying glass.

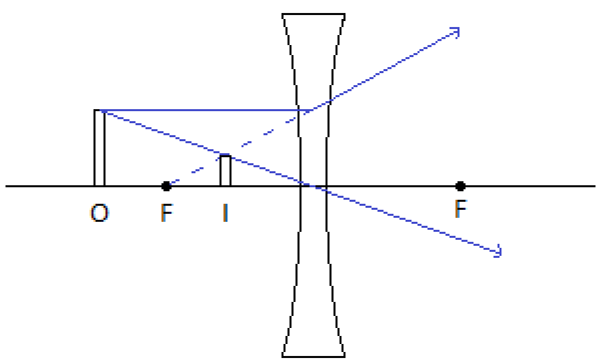


In this situation, the object distance is smallest, so

$$\frac{1}{f} + \frac{1}{d_i} = \frac{1}{d_0}$$

(3) Diverging lens

The third configuration uses a diverging lens. The result is a reduced upright image—regardless of where the object is located. This configuration is used in peepholes, because it allows the viewer (who again is assumed to be located to the right) to see a large object (such as the view of what's outside the door) compressed into a small image.



In this situation, the image distance is the smallest, so

$$\frac{1}{f} + \frac{1}{d_o} = \frac{1}{d_i}$$

Sample problem #955:

Q: A lens makes a 3 cm upright image of a 6 cm object when the object is placed 10 cm from the lens. What is the focal distance of the lens?

A: To answer this question, we must first be able to deduce the lens configuration. Since the image is reduced and upright, this corresponds to configuration (3). A quick sketch will remind us that in this situation, the image distance is the smallest and that we therefore must use the equation

$$\frac{1}{f} + \frac{1}{d_o} = \frac{1}{d_i}.$$

Since the image is half as tall as the object, the image must be located half as far from the lens as the object. Thus, $d_i = 5\ \mathrm{cm}$. Plugging in the numbers and solving for f, we get

$$\frac{1}{f} + \frac{1}{10\,\mathrm{cm}} = \frac{1}{5\,\mathrm{cm}}$$

Which implies that $\boxed{f = 10\,\mathrm{cm}}$.

Evidently, the object is located directly at the focal point!