Chapter 2: Motion along a Straight Line

Kinematics is the study of motion; *dynamics* is the study of the *causes* of motion. Dynamics describes the interaction of objects; kinematics describes what happens in between those interactions

The following quantities are frequently used in kinematics:

displacement $\vec{x}$	distance x
velocity $\vec{v}$	speed v
acceleration $\vec{a}$	magnitude of acceleration a

The quantities on the left are all vectors and must be treated as such. The quantities on the right are equal to the magnitudes (lengths) of the vectors in the left column.

Displacement and distance

Displacement is defined as the position of an object (measured in meters or some other unit of length) relative to some system of axes. Without a set of coordinates to compare to, displacement is meaningless, because there are an infinite number of coordinate systems one could choose. For convenience, it is usually best to just pick the coordinate system that makes the math easiest.

Just like any other vector, displacement has up to three components, but in this chapter we're only going to be dealing with motion in one dimension (i.e. forward and backward).

Distance is defined as the magnitude of displacement. While displacement can be either positive or negative depending on how the axes are set up, distance is always positive.

Velocity and speed

Average velocity is defined as the change in displacement divided by the change in time.

$$\boxed{\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}}$$

where Δ is the Greek letter "delta", which stands for "change in". (Note that Δ is not itself a number, just a modifier of a number.)

If you graph the displacement as a function of time, so that an object moves from position x_1 at time t_1 to position x_2 at time t_2, then its velocity is equal to the slope of the line that connects the two points (see **Figure 1**).

It's important to note that the average velocity doesn't tell you anything about the velocity of the particle *between* those two times. So two racers who finish a race at the same time have the same average velocity over the course of the race even if one racer was well ahead of the other during the majority of

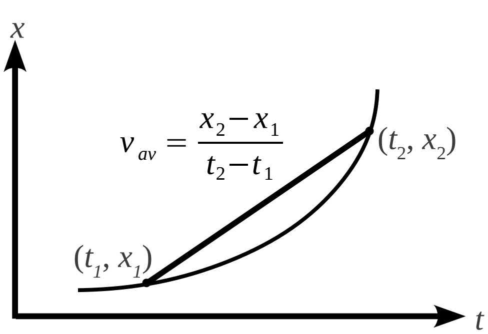

Figure 1: Calculating velocity from a graph

the race.

If you want to find out the velocity at one particular instant, i.e. the *instantaneous velocity*, you must let the change in time become very small. Thus, we zoom in the graph real close to one particular point and look at what the slope is there.

When the change in time is very small, we use the letter *d* in place of Δ. Therefore, the instantaneous velocity can be written:

$$\boxed{v_x = \frac{dx}{dt}}$$

Where symbol $\frac{dx}{dt}$ is called "the derivative of *x* with respect to *t.*"

If you know the equation for position as a function of time, you can figure out the equation for the velocity by using calculus, but we don't need to worry about that right now. Basically you can think of $\frac{dx}{dt}$ as the same thing as $\frac{\Delta x}{\Delta t}$ except that it's evaluated *at* one time instead of *between* two times.

If the velocity is constant (i.e. the speed isn't changing), then the instantaneous velocity and the average velocity are equal.

Just as distance is the magnitude of displacement, *speed* is equal to the magnitude of velocity, and is always positive. The SI unit of velocity and speed is meters per second (m/s).

Acceleration

Just as velocity is the rate of change of displacement with respect to time, *acceleration* is the rate of change of velocity with respect to time.

$$\boxed{a_{av-x} = \frac{v_{2x} - v_{1x}}{t_2 - t_1} = \frac{\Delta v_x}{\Delta t}}$$

An object whose speed is increasing or decreasing is said to be accelerating. Sometimes the term *deceleration* is used to mean a decrease in speed. Because acceleration is a vector quantity, a change in direction of the velocity vector is also considered acceleration—even if the speed stays the same.

There isn't a special name for the magnitude of acceleration, so we just call it by what it is: *magnitude of acceleration.* The SI unit of acceleration is the meter per second per second or m/s^2.

The *instantaneous acceleration* is given by,

$$\boxed{a_x = \frac{d v_x}{d t} = \frac{d^2 x}{d t^2}}$$

which can be evaluated using the calculus.

Graphing kinematic quantities

According to calculus, the graphs of the different kinematic quantities are intimately related to each other. To convert from one to another, we can use the following road-map:

$$\textbf{displacement}\ \underset{\leftarrow\text{area}}{\overset{\text{slope}\rightarrow}{}}\ \textbf{velocity}\ \underset{\leftarrow\text{area}}{\overset{\text{slope}\rightarrow}{}}\ \textbf{acceleration}$$

The slope of the displacement graph is equal to the velocity. The slope of the velocity graph is equal to the acceleration. To go in the opposite direction we add up the area under the curve.

x-t graphs

An *x-t* graph (*Figure 2*) shows the displacement of an object as a function of time. At each point on the graph, the *y*-component represents the displacement, while the *x*-component represents the time.

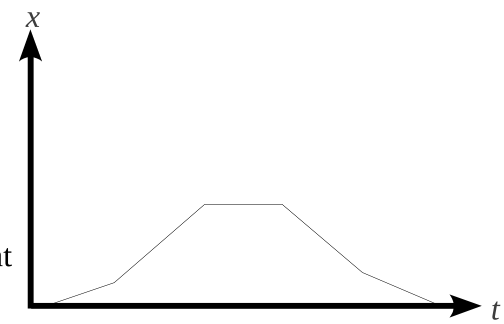

Figure 2*:* *x-t* diagram illustrating a trip to the store and back

Velocity is the slope of the line, so to figure out the velocity at a particular time, we just need to calculate the slope. If the curve is linear, then you can just pick two points, and calculate the slope between them, and that will be the velocity. If the curve isn't linear, then you draw a tangent line to the curve at the particular point you're interested in, and then find the slope of *that* line.

v-t graphs

You can also graph the velocity as a function of time. *Figure 3* shows a *v-t* diagram corresponding to the situation shown in *Figure 2*. When the displacement is increasing, the velocity is positive, when the displacement is decreasing, the velocity is negative.

The graph of the *speed* would be the same except that the second right side of the graph would be above the *t*-axis, as speed cannot be negative.

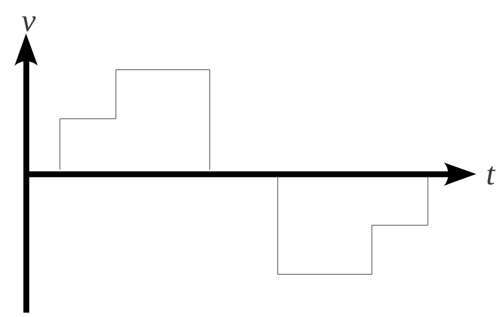

Figure 3: *v-t* diagram corresponding to the situation shown in Figure 2

a-t graphs

Finally, you can graph acceleration as a function of time. *Figure 4* in the next section shows an example of an *a-t* graph.

Constant acceleration

Figure 4 at right shows a particle starting at time $t = 0$ and moving at constant acceleration. If the particle started at rest, then after a time t_1 has elapsed, it will have velocity equal to

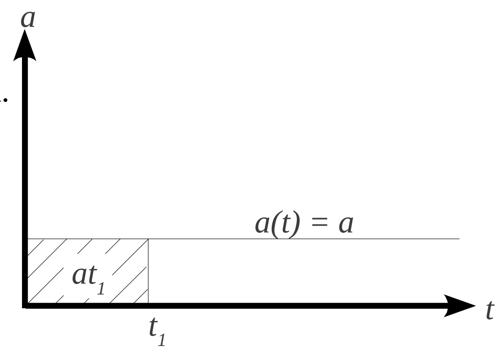

Figure 4: *a-t* graph showing motion under constant acceleration

the area accumulated under the curve: at_1

Or, in general, if the particle started with a velocity of v_0, then the magnitude of its velocity after a time t will be

$$\boxed{v = v_0 + at}$$

In other words, its velocity will increase linearly at a constant rate. This situation is is shown in *Figure 5*.

The area of the shaded region represents the change in displacement between $t = 0$ and $t = t_0$. Using the formula for the area of the trapezoid, we find that $x = v_0 t + ½at_1^2$, assuming the object started at $x = 0$. More generally, if the object started at $x = x_0$, its position at time t is given by

$$\boxed{x = x_0 + v_0 t + \frac{1}{2} a t^2}$$

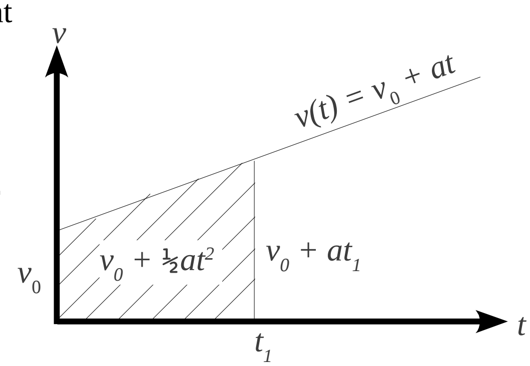

Figure 5*:* *v-t* graph showing motion under constant acceleration

This is the equation of a *parabola* as shown in *Figure 6*.

Notice that if the initial displacement and velocity are both zero, then this simplifies to

$$x = \frac{1}{2} a t^2$$

You can derive another useful equation by solving the velocity equation $v = v_0 + at$ for t and plugging it into the equation for x. This results in:

$$\boxed{v^2 = v_0^2 + 2a\Delta x}$$

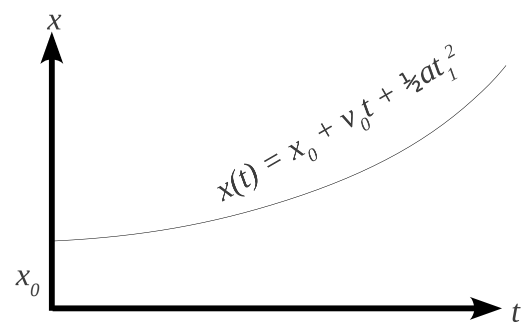

Figure 6: *x-t* graph showing motion under constant acceleration

<u>Free fall</u>

While the equations listed in the last section are very commonly used, it is important to note that they *only* apply to situations where the acceleration is constant. The most familiar situation in which objects undergo constant acceleration is free fall. Other cases in which constant acceleration occurs include a train pulling out of a station, an object coming to a stop as it slides along a frictional surface, a car braking, or anything in which the forces (see Chapter 4) acting on an object do not change.

Free fall is what happens when an object is placed in a gravitational field and no forces act on it other than gravity. The acceleration produced during free fall depends on the mass and the distance of the object which is causing the gravity. The gravitational acceleration at the surface of the Earth (abbreviated g) is about 9.80 m/s^2, but on Mars, where the gravity is weaker, the acceleration is only about 3.7 m/s^2.

If one were to rise to an altitude of 400 km above the Earth's surface (the height at which the space shuttle orbits) the acceleration would drop to about 8.7 m/s^2. (We will learn how to predict gravitational acceleration at varying altitudes in Chapter 12).

The following table shows the displacement, velocity and acceleration of a falling object near the surface of the Earth. (Note that for clarity I have used 10 as an approximation for 9.8.)

Time (s)	Acceleration (m/s^2)	Velocity (m/s)	Displacement (m)	Change in displacement (m)
0	-10	0	0	
1	-10	-10	-5	-5
2	-10	-20	-20	-15
3	-10	-30	-45	-25
4	-10	-40	-80	-35
5	-10	-50	-125	-45
6	-10	-60	-180	-55

The thing to note here is that during each second the speed increases by a constant amount which numerically equal to the gravitational acceleration, while the distance increases at an ever increasing rate.