Chapters 4-5: Newton's Laws of Motion

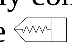
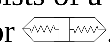
In Chapters 2 and 3, we studied *kinematics*, which is the language for describing motion. In Chapters 4 and 5, we introduce *dynamics*, the relationships between motion and its causes. In addition to the quantities in the last two chapters, we need to add two more: *force* and *mass*. Mass is amount of matter in a substance. It is measured in kilograms. Mass is a scalar quantity. For most purposes, mass is a constant, though we may consider a few objects—such as rockets—that have changing mass.

Forces

A *force* is defined as a push or a pull. The SI unit of force is the Newton. To give you an idea of the scale of a Newton, the force of gravity on an apple is about one Newton, while the weight of a 150 lb human is about 670N. Many impact forces vary in strength with respect to time. A boxer may be able to exert a peak force of 5000N. A baseball bat can exert an average force of as much as 30 or 40 kN, but only for a very brief time.

Like displacement, force is a vector quantity, so if more than one force is acting on a body, the *net force* is the vector sum or *resultant* of the forces. We can represent the vector sum of forces $\vec{\boldsymbol{F}}_{net}$ or by $\sum \vec{\boldsymbol{F}}$ where Σ is the (capital) Greek letter "sigma". As with displacement, it is often convenient to break a force vector into its components. When we do this, we put an 'S' symbol through the original force so we don't get confused and think that there are actually three forces.

Some forces are *contact forces*, involving direct contact between the bodies. Others are *long-range forces*, meaning that they can act even when the bodies are separate.

Forces can be measured in a variety of ways. Weight can be measured with a scale, and a tension in a cord can be measured with a spring scale, which basically consists of a frame with a fixed spring inside it attached to a hook. You will see them represented like  or .

Newton's Laws

The principles of dynamics can be summarized by three short statements known as *Newton's Laws of Motion*. They are as follows:

1. When no forces are acting on an object, the object's motion does not change (i.e. it travels in a straight line with constant velocity and no acceleration.)
2. When the net force acting on an object is *not* zero, it has an acceleration which is proportional to the net force vector and inversely proportional to its mass.
3. Whenever two bodies interact, the force of the 1^{st} on the 2^{nd} is equal to the negative (opposite) of the force of the 2^{nd} on the 1^{st}.

We will look at each of these laws in more detail as we go along

Newton's First Law

Newton's First Law states that *when an object is acted on by no forces its acceleration is zero and its velocity remains constant*. In particular, if the object is at rest (i.e. its velocity is zero) it will stay rest. The tendency of objects to stay in motion once set in motion is called inertia, and hence Newton's first law is also known as the *Law of Inertia.* All objects with mass have inertia.

Newton's First Law also holds when forces are applied to an object if the net force equals zero. Such an object in is said to be in *equilibrium.* Thus zero net force is the same as no force at all.

Throughout all this, we have been modeling the object as a point particle with zero size. If the object has greater than zero size, then the force can hit the object off-center. This wouldn't violate any of Newton's laws, but it would cause the object to rotate, which we are going to ignore until Chapter 10.

Inertial reference frames

Newton's First Law is not valid in all reference frames. For example, consider what would happen if you were to stand in the passageway of an airplane while the plane was accelerating. You would tend to stay at rest due to the Law of Inertia, and therefore, you would move backwards relative to the plane. To other passengers in the plane, there would seem to be a force acting on you pushing you toward the back of the plane, even though there is no force.

The reason that Newton's First Law does not hold in this case is that for the passengers, the reference frame *itself* is accelerating. Because accelerating reference frames are not moving inertially, they are called *non-inertial*. Newton's First Law is valid only in *inertial reference frames*.

Real and fictitious forces

The force that seemed to be pushing pushing you toward the back of the plane is an example of a fictitious force. The force that accelerates you into the back of your seat when you step on the gas pedal is another. Nothing is actually pushing you. Instead the reference frame is accelerating toward you. However, because you are inside the car, it is natural to think of the car as an inertial even though it isn't.

Another example of a fictitious force is the centrifugal force. The centrifugal force is the force that seems to push you outward when you run in a circle. However, there isn't any *real* force pushing you outward, it is just your inertia tending to move you in a straight line. The faster you go, the harder you must push (apply a centripetal force) to keep yourself moving in a circle.

Another example of a fictitious force is the Coriolis force. This is the force that causes wind currents in the northern hemisphere to rotate clockwise and wind currents in the southern hemisphere to rotate counter-clockwise. It is not the winds themselves that are moving this way, but the fact that the Earth is rotating underneath them.

Just because fictitious forces are called fictitious doesn't mean they can't cause damage. A fictitious force can kill you if it is strong enough.

Newton's Second Law

Newton's First Law tells us that objects acted on by forces undergo acceleration, but it does not tell us what that acceleration is. Newton's Second Law does that. It states: *the acceleration of an object is proportional to the net force acting on it (measured in Newtons) and inversely proportional to its mass.*

In other words, if you double the force while keeping mass constant, the acceleration will be twice what it was before, and if you double mass while holding force constant, the acceleration will be half what it was before.

Mathematically this is written

$$\vec{a} = \frac{\Sigma \vec{F}}{m}$$

But more often you will see it written as

$$\Sigma \vec{F} = m\vec{a}$$

If you hit two objects with the same force and one is more massive than the other, the more massive one will accelerate less. The greater the mass, the greater the resistance to acceleration. Therefore, mass is a quantitative measure of inertia.

The reason that all objects fall at the same rate when dropped from a height is due to Newton's Second Law. Take two balls of identical properties except that one has twice the mass of the other. The smaller ball will fall at a rate equal to the force of gravity divided by its mass. because the second ball is twice as massive, the denominator in the acceleration expression will be multiplied by two; however, since gravity is proportional to mass, the numerator in the expression will also be twice as large. Therefore, the twos will cancel out and both balls will fall at the same rate. This rate is called *g*, and on Earth it is equal to -9.8 m/s^2.

Weight

Weight is the name given to the force on an object due to gravity. According to Newton's Second Law, the weight of an object in a gravitational field is equal to its mass multiplied by its acceleration.

$$\vec{W} = m\vec{g}$$

The reason that all objects accelerate at the same rate when dropped is due to Newton's Second Law. Take two balls of identical properties except that one has twice the mass of the other. The smaller ball will fall at a rate equal to the force of gravity ($W = mg$) divided by its mass (m) which equals g. Because the second ball is twice as massive, the denominator in the will be twice as large; however, since gravity is proportional to mass, the numerator will also be twice as large. The twos will cancel out and both balls will fall at the same rate.

On Earth, the value of g is 9.8 m/s^2. On the surface of a less massive planet—like Mars—the gravitational acceleration is less (about 3.7 m/s^2). At great altitudes, the gravitational acceleration starts to drop off. At 100km—the altitude the International Space Station orbits at—is only about 8.7 m/s^2. We will learn how to calculate these values in Chapter 12.

Newton's Third Law

Newton's Third Law states that whenever two objects are in contact with each other, the force of one object on another (F_{AB}) is equal to the force of the second object on the first (F_{BA})—but in the opposite direction.

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

When I punch the wall, the reason my hand hurts afterwards is because in addition to the force I exert on the wall, there is also and equal and opposite force exerted by the wall on my hand.

You might be thinking, since all the forces are equal and opposite to each other, why don't they cancel each other out—and if they always cancel each other out, how can anything move at all? The answer is that the two forces are acting on *different objects*. The forces acting on each *individual* object are not necessarily balanced, and hence they can move.

When you take a step, you exert a force on the earth, and the earth exerts an equal force back on you, and you both acceleration in opposite directions. It may seem strange that the force exerted by you on the earth is equal to the force exerted by the earth on you. Because the earth is so much larger, shouldn't its force be larger? But according to the Third Law, forces have to go both ways. Every particle in the earth is pulling on you, and therefore you are pulling on every particle in the earth.

There is nothing in the third law, however, that says the accelerations have to be equal. You don't notice the acceleration of the Earth is that because the Earth is so much more massive that its acceleration according to $a = F/m$ is practically zero. You, on the other hand, have a much smaller mass and accelerate at 9.8 m/s^2.

<u>Normal force</u>

The normal force (symbolized N or F_N) is a force caused when two surfaces come in contact with each other. Assuming the surface is hard and uncompressible, the normal force will always be equal to the force applied perpendicularly to the surface. (The term "normal" in mathematics is synonymous with perpendicular, so the force always makes a 90 degree angle with the surface.) If the surface is slanted, the normal force will have a horizontal as well as a vertical component (see Figure 1, right). The normal force is caused by electrons in the surface which repel each other when pushed together by forces acting on them. If the force is large enough, the it is possible that the resistance of the atoms to motion may be insufficient to resist motion, and then surface will either bend or break.

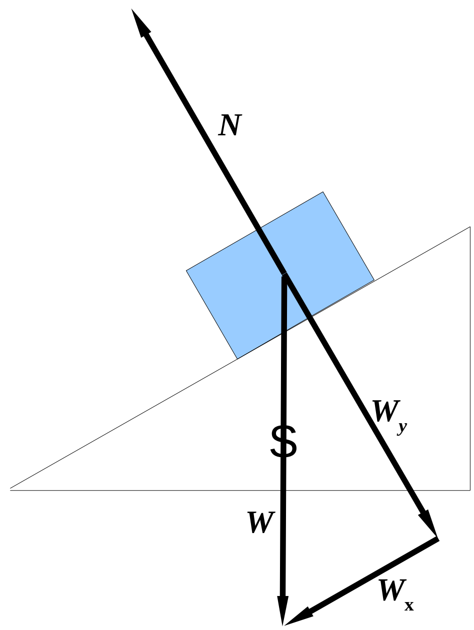


Figure 1: Force diagram of a block sliding on a (frictionless) inclined plane. (The S symbol indicates that the **W** vector has been replaced by its x- and y-components.

<u>Friction</u>

Friction is the force which is caused by the horizontal sliding motion of two surfaces relative to each other. Whenever two objects are in contact, there will invariably be small imperfections in their surfaces that will get caught in each other when they slide, and any force that causes them to slide relative to each other will be resisted by electrons in the bumps repelling each other. Generally, the friction force is be greatest when the area of contact is large, the bumps are relatively tall, and the forces pushing the two surfaces together are large. If it wasn't for friction, you wouldn't be able to walk, because you wouldn't be able to grip the ground to push off. Hiking boots have thick soles with many bumps in them in order to create as much friction as possible when standing on a rocky surface.

Although frictional forces can be used to initiate motion, as we just saw, in this book we will generally restrict the term "friction" to mean only forces which tend to prevent or slow down motion—such as a car that slowly rolls to a stop when the driver's foot is taken off the accelerator.

Friction can be divided up into two types: static friction and kinetic friction. If the force applied to an object is not enough to overcome the frictional repulsion between atoms in the surfaces, then the friction is termed *static friction.* As with the normal force, the static friction will always be equal and opposite to the force being applied.

$$F_{\text{fr}} = -F_{\text{applied}}$$

If the applied force is increased, eventually the static friction will give way and the object will begin to slide (see Figure 2, right). The *maximum* force exerted by static friction is proportional to the normal force and is given by

$$F_{\text{fr}} = \mu_s N$$

Where μ_s is a parameter known as the *coefficient of static friction.* The rougher the surface, the greater the coefficient of static friction will be. A table of coefficients of friction for various surfaces is given in Table 4-2 on page 90.

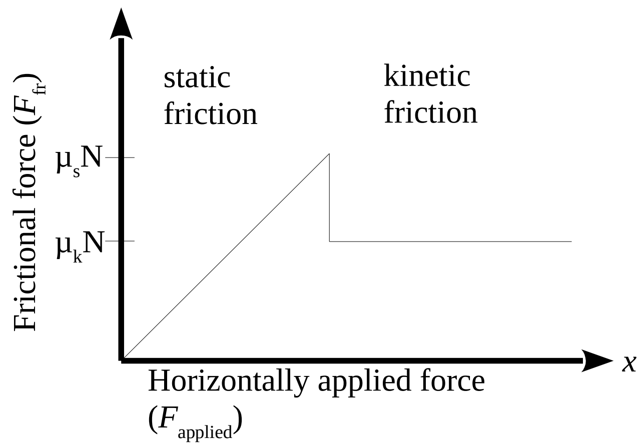


Figure 2: The static friction force is equal and opposite to the applied force until the the force equals $\mu_s N$. Once the object starts moving, the force is constant and equals $\mu_k N$.

Once the object is moving, the friction is known as *kinetic friction*. Kinetic friction always acts in a direction opposite the direction of motion (not necessarily the direction of net force) and is also proportional to the normal force.

$$F_{\text{fr}} = \mu_k N$$

The *coefficient of kinetic friction* μ_k is generally less than the coefficient of static friction, because moving objects can "skim" across the surfaces they are resting on.

Tension

Tension (symbolized T or F_T) is a force that tends to pull apart or stretch a cable or rope. In physics problems, it usually assumed that ropes are weightless and do not stretch or bend due to the forces acted upon them. Therefore, the tension in any particular cable is considered to be the same at every point, and when objects are connected by cables, forces are transmitted undiminished from one object to the other. At every point along a cable, there are two tensions directed in opposite directions which cancel each other out.

Free body diagrams

When solving physics problems, it is often convenient to draw a figure showing all forces acting on a particular object. Such a diagram is called a *free-body diagram* or a *force diagram.* In a free-body diagram, the object is represented by a point at the origin, and the forces are represented by outward-pointing vectors (often broken into components) with labeled magnitudes and angles. It is important that one only includes forces acting on the object in question, because forces acting on other objects have no direct influence on the object's motion.

<u>Steps for solving Newton's Second Law problems</u>

1. Draw a simple sketch of situation labeling forces.
2. Draw a free-body diagram for each object of interest, showing all the forces acting on that object.
3. Choose appropriate *x* and *y* coordinates and define the positive direction for each
4. Determine the components of each force acting in the *x* and *y* directions.
5. Write Newton's Second Law for each component of each object.
6. Rewrite each equation in terms of known quantities.
7. Confirm that there are enough equations to solve for each unknown and solve them!

<u>Fluid resistance</u> (optional)

Fluid resistance is the resistive force felt by objects that are moving through a liquid or a gas. Like surface friction, fluid resistance is directed opposite to the direction of motion; unlike surface friction, which is generally velocity independent, fluid resistance increases with speed.

In gases, where inertial forces are typically much greater than resistive forces (i.e. viscosity is low), the resistance is proportional to the square of the speed:

$$f = D v^2$$

Here, *D* is a quantity known as the *drag coefficient*.

In liquids, where the viscosity is much higher, the resistance force is proportional to the first power of the speed:

$$f = k v$$

The constant *k* is similar to the drag coefficient, but doesn't have any special name (that I know of).

In either case, an object which is acted on by both gravity and fluid resistance forces will accelerate until its weight and the resistance force become equal. When this happens, the object will be in equilibrium, so the speed will be constant. This speed is called the *terminal speed* or *terminal velocity.* (Note that because the force changes with speed, bodies experiencing fluid drag do *not* undergo constant acceleration.)

It can be shown that in low viscosity situations (such as air drag), the terminal speed is given by

$$v_t = \sqrt{\frac{m g}{D}}$$

while in high velocity situations (like sinking through water), the terminal velocity is:

$$v_t = \frac{m g}{k}$$

<u>Fundamental forces</u>

At a fundamental level, there are exactly four kinds of forces known: electromagnetism, gravity, the strong nuclear force and the weak nuclear force. These are all long-range forces according to the above definition, but that effective range may be anywhere from 10^{-15} m to infinity.

All of the everyday forces like weight and friction and normal pushes and pulls on objects are all examples of either electromagnetism or gravity. The other two, because of their extremely short range are relevant mainly in nuclear physics. Each of the fundamental forces has a characteristic strength. Not surprisingly, the strong force is the strongest. Gravity is the weakest. To give you a sense of how weak the gravitational force is, consider that the electrostatic repulsion between the electrons in my feet and the electrons in the ground is strong enough to withstand the gravity of the *entire* Earth.