Chapter 6-7 Notes

Work

Work in physics is defined as

$$\boxed{W = \vec{F} \cdot \vec{s} = F\, s \cos\theta}$$

where *F* is a force and *s* is the distance over which it acts. If the force is directed at an angle *θ* relative to the displacement, then only the component of the force which is in the same direction as the motion actually produces work. This component is denoted $F_{||}$.

If the force has a component in the same direction as the displacement, the work is positive; if the force has a component in the opposite direction of the displacement, then the work is negative. If the force is entirely in the same direction as the displacement ($\theta = 0°$), then the work equation becomes simply

$$W = F\, d$$

If the force is entirely perpendicular to the displacement ($\theta = 90°$), then work is equal to zero. Therefore, the normal forces and centripetal forces never do any work.

If the force varies with respect to position, then the work equation becomes.

$$W = \int_{x_1}^{x_2} \vec{F} \cdot d\,\vec{x} = \int_{x_1}^{x_2} F \cos\theta\, dx$$

The SI unit of work is the *joule*, which is abbreviated "J". One joule is equal to one newton times one meter. In the British system of units, the unit of energy is the *foot-pound* and is equal to one foot multiplied by one pound.

Energy

The *work-energy theorem* states that, for all conservative forces (see below), work equals the change in a quantity called *energy*.

$$\boxed{W = \Delta E}$$

When a force is does work on an object, the energy of the object increases by an amount equal to the work; when an object does work, its energy decreases by an amount equal to the work.

Nobody knows exactly what "energy" *is.* Energies themselves can (usually) be measured only relative to some "standard" value; only a *change* in energy can be measured directly. Quantities that behave in this way are called *state functions.*

There are many different forms of energy. When deriving the formula for an unknown form of energy, one first calculates the work and then rewrites the work equation as a change in some quantity; this quantity is the sought-after energy.

Kinetic energy

Kinetic energy is the energy due to an object's speed. If there is only one force acting on an object,

$$W = F_{\text{net}} d = m a d$$

According to equation 2-10,

$$a = \frac{v_2^2 - v_1^2}{2d}$$

Substituting this in to the work equation, we get

$$W = m\left(\frac{v_2^2 - v_1^2}{2}\right) = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

Therefore, the *kinetic energy* of an object is given by

$$\boxed{K = \frac{1}{2} m v^2}$$

Notice that it is not specified what the velocity is measured *relative* to. A moving car has zero kinetic energy relative to a passenger in the car, but has high kinetic relative to a pedestrian on the street. On the other hand, the pedestrian has low kinetic energy relative to the street, but high kinetic energy relative to the passenger in the car.

Potential energy

The energy due to change in height is known as *gravitational potential energy.* The term “potential” is used because although this kind of energy is not due to motion (like kinetic energy), it can be converted into kinetic energy. It can be calculated as follows:

$$W = Fd = mgd = mg(h_2 – h_1) = mgh_2 - mgh_1$$

Therefore, the equation for gravitational potential energy is $\boxed{U = mgh}$

There are several other kinds of potential energy. The potential energy stored in a compressed spring can be released to cause motion. The energy of chemical bonds holding together electrons in a molecule can be considered potential energy as can the energy holding protons together in a nucleus. The potential energy stored in these bonds is the fuel for chemical and nuclear reactions, respectively.

The following table lists the equations for several kinds of forces, and their corresponding energies.

Quantity	**Force**	**Energy**
Speed	ma	$\frac{1}{2} m v^2$
Gravity (uniform field)	mg	mgh
Gravity (generalized)	$\frac{G m_1 m_2}{r^2}$	$-\frac{G m_1 m_2}{R}$
Friction (non-conservative)	$\mu_k m g$	$\mu_k m g x$
Elasticity (spring)	$-kx$	$\frac{1}{2} kx^2$

Power

Power is defined as the amount of work done divided by the time it takes for the work to be done:

$$\boxed{P = \frac{W}{\Delta t}}$$

Since $W = F\Delta d$, this can be rewritten as

$$P = F \frac{\Delta d}{\Delta t} = F v$$

In other words, the power can be thought of as the force required to maintain an object moving at a constant speed multiplied by the speed. Power is measured in *watts;* one watt is equal to one joule per second.

The *efficiency* of a process is the amount of useful power output divided by the amount of power input.

$$e = \frac{P_{\text{out}}}{P_{\text{in}}}$$

Efficiency is always less than 100%, because some energy is always converted into non-useful forms such as thermal energy or friction. This is a result of the second law of thermodynamics.

Conservation of energy

The *conservation of energy* is the fact that whenever you calculate the energy before and after some event has occurred, it always has exactly the same value.

There are several ways of expressing the law of conservation of energy:

1) *Global interpretation:* The total amount of energy in the universe is constant

$$\Delta E_{universe} = 0$$

While correct as far as anybody knows, this statement of the law doesn't have much practical utility.

2) *Spatial interpretation:* If the universe is divided the universe into two parts, (referred to as the "system" and the "surroundings"), the energy gained by one must equal the energy lost by the other.

$$\boxed{\Delta E_{\text{system}} = -\Delta E_{\text{surroundings}}}$$

Energy may be transferred through the direct movement of mass from the system to the surroundings (or vice versa) or if the system does work on the surroundings (or vice versa). If no material objects may move from one to the other, then the system is said to be *closed*. If no energy *at all* can be exchanged between the system and its surroundings, then the system is said to be *isolated* and $\Delta E = 0$. There is also a third form of energy transfer known as *heat*, which includes all other forms of energy transfer that are not one of the other two (see Chapter 17).

3) *Temporal interpretation:* The amount of energy in the system after an event is always equal to the energy in the system before the event minus the energy lost by doing work on its surroundings.

$$\boxed{E_f = E_i - W}$$

In an *isolated system*, no energy is transferred, so the work is equal to zero, and this equation becomes

$$E_f = E_i$$

Conservative forces

A *conservative force* is a force for which the work done in moving an object from one point to another is *path independent;* in other words, the work depends only on the initial and final positions. Anything that happens to the object in between these positions will not affect the total work done by this force.

Systems that are acted upon only by conservative forces are *reversible*: returning the object to its original position will result in the recapture of the original energy in its original forms. Gravity, elasticity and electricity are conservative forces. Tension, friction and fluid resistance are all non-conservative forces.

Mechanical energy is defined as the sum of kinetic energy and potential energy of an object. If all forces acting on a system are conservative, then the mechanical energy is conserved.

$$\Delta K + \Delta U = 0$$

If not all forces are conservative, then some energy is converted into *internal energy*, causing the temperature of the system or surroundings to increase. Careful experiments have shown that the change in internal energy is exactly equal to the work done by nonconservative forces. Thus,

$$\Delta K + \Delta U + \Delta U_{\text{int}} = 0$$