- When we calculated the gravitational potential energy in a previous chapter, we assumed that *g* was a constant. To calculate the gravitational potential energy over large distances, we have to take into account the fact that *g* can vary. Therefore, we have to start with the more general definition of gravitational force.

$$
\begin{aligned}
W &= \int_{R_1}^{R_2} \frac{G m_1 m_2}{R^2} dR \\
&= G m_1 m_2 \int_{R_1}^{R_2} \frac{1}{R^2} dR \\
&= \left[\frac{-G m_1 m_2}{R} \right]_{R_1}^{R_2} \\
&= \left(\frac{-G m_1 m_2}{R_1} \right) - \left(\frac{-G m_1 m_2}{R_2} \right)
\end{aligned}
$$

- Therefore the potential energy due to a generalized gravitational field is given by

$$
-\frac{G m_1 m_2}{R}
$$

- Notice that the result is negative. This makes sense, because the potential energy is lowest (most negative) when the objects are closest together and highest when the distance between them is great. If zero potential energy is set to zero when $R_1 = \infty$ (far enough apart that the first term becomes negligible), the change in energy becomes simply.

$$
\frac{G m_1 m_2}{R}
$$

- Because the potential energy at the surface of a planet is negative (relative to an object in outer space free from gravitational interactions), it is often said that any object located in a gravitational field is in a potential "well".

- Suppose we launch a projectile with the intention of sending it into space. Such a projectile when it is near the surface of the Earth would have to have a lot of velocity; when it reaches outer space ($R = \infty$), it will have a lot of gravitational potential energy, because by falling into the gravitational well from that distance would result in the acquisition of a lot of kinetic energy.
    - According to the law of conservation of energy, we can say that the kinetic energy lost while gaining altitude is equal to the potential energy gained

$$
\frac{1}{2} m_O v_i^2 = G \frac{m_O m_E}{R}
$$

Equivalently, we could set the gravitational potential energy in deep space equal to zero and write that that the total initial energy (partly kinetic, partly potential) equals the total final energy (which is zero because the projectile is far from any gravitational source and all its initial velocity has been "used up" in the act of escaping)

$$-G\frac{m_O m_E}{R_E}+\frac{1}{2}m_O v_i^2=0$$

Either way, solving for v_i, we see that

$$v_i=\sqrt{\frac{2Gm_E}{R_E}}$$

- This velocity is called the *escape velocity.*
- A common misconception is that a rocket fired from the Earth's surface must reach the escape velocity in order to escape orbit. But this is not true because rockets do not start out with a constant velocity, but continue to accelerate throughout their altitude gain. The escape velocity is the velocity a *projectile* would need to have to escape Earth's orbit.
- In reality a rocket could not go at speeds near the escape velocity near the surface of the Earth because air resistance would cause it to burn up. Speeds like this are normally seen by spacecraft only after they have reached space, or during reentry.
- Escape velocity depends on distance from the object is escaping from. For an object already in orbit around the Earth, the escape velocity is less, because it the work integral is smaller. The escape velocity from the sun is about 617 km/s at the surface of the sun but only about 42 km/s from the orbit of the Earth.
- Rockets and space elevators avoid the need for such high velocities because they are not projectiles. In order to continue rising, such an object needs only to have a vertical force greater than gravity at each point.