Chapter 29: Electromagnetic induction

So far we've talked about electrostatics: what happens when there is an unmoving configuration of charge resulting in unchanging electric fields, but nothing else.

Then we talked about what can be called magnetostatics: where there charges are allowed to move. This results in an unchanging *current* which produces an unchanging magnetic field. So even though charges are moving, their rate of change is fixed.

Now we're going to talk about what happens when we allow the currents to change. This results in changing magnetic fields. And it turns out that when magnetic fields change, you get an electric field. And if this changes, it can induce new magnetic fields, which in turn induce new electric fields and so on. In other words, electric and magnetic fields become intricately linked.

The process by which a changing magnetic field produces an electric field is called *electromagnetic induction*. Electromagnetic induction is used in to produce the vast majority of the electricity that people use through a device called a generator, which we will talk about.

Another familiar example of electromagnetic induction is a credit card reader. The credit card has its information encoded in the form of magnetized patterns. When you swipe it through the reader, it results in a changing magnetic field, which in turn results in an electric field, which produces a changing current, which the machine then converts into ones and zeros.

Let's look at a stripped down version of this card reader. It consists of just a coil of wire connected to an ammeter and a bar magnet. When the north pole of the magnet is moved near the coil, a current is detected in the ammeter. When it is moved away a current is detected in the opposite direction.

If instead of a magnet, we use another coil with a current running through it, the same thing happens because a current loop produces a magnetic field just like a magnet does.

To take it one step further, let us consider a coil of wire placed in between the poles of an electromagnet as shown in fig. 29.2.

When the electromagnet is off, there is no current detected. When the current is turned on, there is a brief period of time in which a current is detected and then as the magnetic field levels off to a constant intensity, the current returns to zero.

If the current loop is squeezed so that its cross sectional area is reduced, then a current is detected during the deformation, but not before or after. Then if we return the coil to its original size, we observe a current flowing in the opposite direction.

If we *rotate* the wire coil so that the number of field lines flowing through it decreases, we detect a current during the rotation.

If we remove the coil from the loop, we detect a current. If we unwind one of the turns in the coil, we detect a current.

For all of these changes, the faster the rate at which they are carried out, the greater the current, and the greater the resistance in the coil, the lower the current. This shows that the electromagnetic force doesn't depend on the material.

In general, anything that changes the flux through the coil produces a current.

$$E = \frac{-d\Phi_B}{dt}$$

Where Φ_B is the magnetic flux through the coil. (This is called *Faraday's law*.) The minus sign is there because of a convention. While it's possible to figure out the sign of the induced emf using conventions, it's usually easier to just calculate the magnitude using Faraday's law and figure it out the direction using something called *Lenz's law*. Lenz's law states that the induced emf always tends to produce a magnetic field which points in the opposite direction of effect that created it.

[Example 29.2]
We can see that if the field is decreasing, the effect would be to decrease the flux through the coil going to the right, or increase the flux through the coil going to the left, so the direction of the effect is to the left. [Draw vectors on board.]

Therefore, according to Lenz's law, the induction will tend to produce an increased magnetic field pointing to the right. Therefore, the current must flow clockwise as we look through the coil towards the right.

(Note that Lenz's law doesn't say that the induced field will cancel out the original one, just that it will point in the opposite direction; the actual magnitude of the induced current depends on the resistance in the coil. If the coil had no resistivity, the any change to field inside it would be completely canceled out, but if the coil was made out of wood, the induced current would be practically zero.

[Example 29.3: The search coil]

You can use the electromagnetic induction to measure the Earth's magnetic field.

$$E = \frac{-d\Phi_B}{dt} = -NBA\cos\theta$$

$$\int_0^T E\,dt = -NBA \int_{\pi/2}^{3\pi/2} \cos\theta d\theta$$

$$\int_0^T IR\,dt = -NBA\left[\sin\theta\right]_{\pi/2}^{3\pi/2}$$

$$R\int_0^T I\,dt = 2NBA$$

$$\frac{R\int_0^T I\,dt}{\Delta t} = \frac{2NBA}{\Delta t}$$

$$B = \frac{I_{avg} R \Delta t}{2NA}$$

[Example 29.4: An alternator]
[Example 29.6: emf in a slidewire generator]
[Example 29.7: work and power in a slidewire generator]

When you try to move the rod, the induced current produces a force F = ILB which opposes the direction of motion (as it must according to Lenz's law). Thus, in order to for the generator to generate current, one must apply a force at least as large.

It's important to note that the magnetic force itself doesn't do any work. The rod as a whole is neutral, so can't be affected by magnetic forces. Instead, the

Motional electromotive force. Another electromagnetic induction effect happens when a conducting object is moved through a magnetic field. [Fig. 29.4]

$$E = vBL$$

In terms of vectors, this is:

Section 29.5: Induced electric fields

Although we've mostly been concerned with induced *emfs* in wires so far, it's important to realize that the more fundamental effect of a changing magnetic field is to produce an electric field. The currents generated are there even if there's no matter for the field to penetrate. And we know it isn't magnetic forces causing the emf, because you can create one when the wire isn't even *in* a magnetic field [Fig. 29.16].

An important thing to notice about this *induced* electric field is that it is non-conservative. Its electric field lines form a loop which returns to their starting point, so that a charge which makes a complete circuit would have a *lower* potential than it had when it started according to $V = \int_a^b \vec{E} \cdot d\vec{l}$.

We can rewrite Faraday's law to reflect this plugging this equation into Faraday's law to get the more general equation

$$\int_a^b \vec{E} \cdot d\vec{l} = \frac{-d\Phi_B}{dt}$$

We can use this equation to calculate the magnitude of induced electric field at a point on the loop in fig. 9.16:

$$E = \frac{1}{2\pi r}\left|\frac{d\Phi_B}{dt}\right|$$

Where the direction of the field is given by Lenz's law.

Generalized Ampere's law

In chapter 28, we derived Ampere's law assuming that the currents (and therefore the electric fields were constant). However, if you allow changing electric fields, then Ampere's law is incomplete. For example, if you try to apply Ampere's law to a current flowing into a capacitor, you run into a contradiction. [draw Fig. 29.21]

Ampere's law doesn't state any particular rule about *where* the enclosed current passes through the integration loop, only that it must. So if we add up the currents passing directly through the plane containing the loop, then we get a current. However, if we stretch the surface so that it goes between the capacitor plates, then *no* current flows through the loop.

However, we know that current is equal to the time derivative of charge, so if we calculate the charge built up on a capacitor plate as follows:

$$q = Cv = \frac{\epsilon A}{d} Ed = \epsilon EA = \epsilon \Phi_E$$

Then we can invent something called the "displacement current"

$$\frac{dq}{dt} = \epsilon \frac{d\Phi_E}{dt}$$

Which has the same units as current, but doesn't involve an actual movement of charge.

To distinguish it from the real current in the wire, we call the displacement current i_d and the real current i_c.

Then if we rewrite Ampere's law to include the displacement current, we get

$$\oint \vec{B} \cdot d\vec{l} = \mu_o (i_C + i_D)$$

or

$$\oint \vec{B} \cdot d\vec{l} = \mu_o \left(i_C + \epsilon \frac{d\Phi_E}{dt} \right)$$

Therefore the magnetic field produced inside a capacitor is given by… [fig. 29.22]