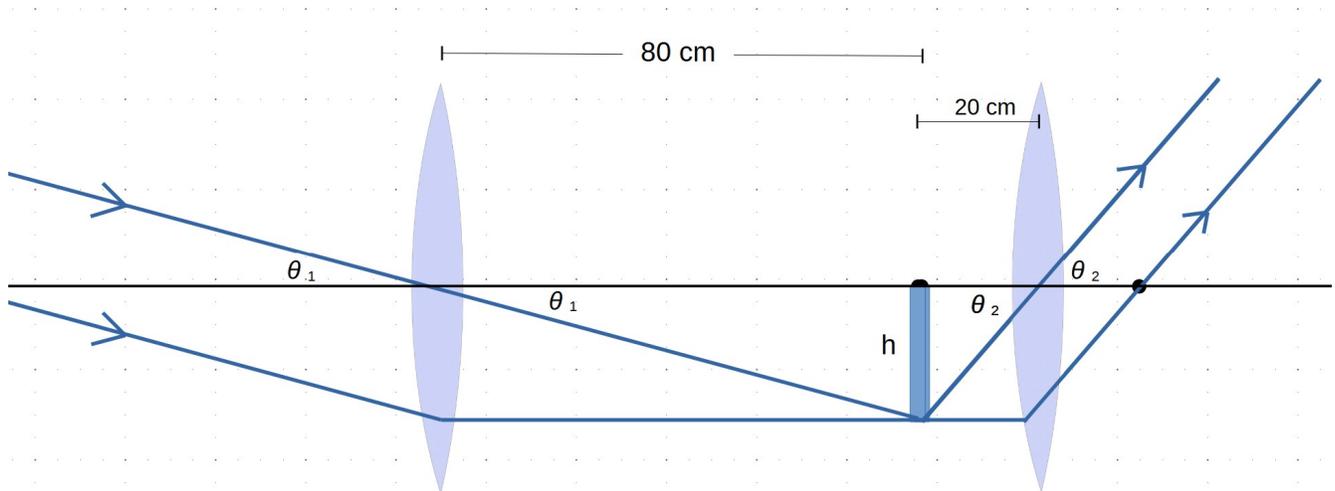


Q: A refracting telescope consists of two converging lenses separated by 100 cm. The eye-piece lens has a focal length of 20 cm. Find the angular magnification of the telescope.

A: To construct a telescope from two converging lenses, one may arrange for the image of the first lens to coincide with the focal point of the second lens. This will result in a virtual image at infinity.



The angular magnification is given by

$$M = \frac{\theta_2}{\theta_1} = \frac{\tan^{-1}\left(\frac{h}{20}\right)}{\tan^{-1}\left(\frac{h}{80}\right)}$$

Assuming that the angles are small (i.e. paraxial approximation), then we can say $\tan^{-1} x \approx x$

such that this expression simplifies to

$$M = \frac{\frac{h}{20}}{\frac{h}{80}} = \boxed{4}.$$

Transfer matrices

Another way to approach this is with optical transfer matrices. We represent the ray that intersects the first lens at the optic axis by the vector

$$\vec{v}_1 = \begin{bmatrix} 0 \\ \theta \end{bmatrix}$$

where the first component is the height of the ray relative to the optic axis and the second is the angle.

We left-multiply this by the lens matrix

$$L_1 = \begin{bmatrix} 1 & 0 \\ -\frac{1}{80} & 1 \end{bmatrix}$$

As expected the vector is unchanged:

$$\vec{v}_2 = L_1 \vec{v}_1 = \begin{bmatrix} 1 & 0 \\ -\frac{1}{80} & 1 \end{bmatrix} \begin{bmatrix} 0 \\ \theta \end{bmatrix} = \begin{bmatrix} 0 \\ \theta \end{bmatrix}$$

To propagate this vector from the first lens to the second, we multiply by

$$P = \begin{bmatrix} 1 & 100 \\ 0 & 1 \end{bmatrix}$$

resulting in

$$\vec{v}_3 = P \vec{v}_2 = \begin{bmatrix} 1 & 100 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ \theta \end{bmatrix} = \begin{bmatrix} 100\theta \\ \theta \end{bmatrix}$$

Finally, we multiply by the second lens matrix (focal length = 20 cm) to get

$$\vec{v}_4 = L_2 \vec{v}_3 = \begin{bmatrix} 1 & 0 \\ -\frac{1}{20} & 1 \end{bmatrix} \begin{bmatrix} 100\theta \\ \theta \end{bmatrix} = \begin{bmatrix} 100\theta \\ -4\theta \end{bmatrix}$$

Evidently the magnitude of the angle has increased by a factor of 4.